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Nonlinear Propagation of Aircraft Noise in the Atmosphere

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Carefully controlled tests of aircraft noise propagation have shown instances of anomalously low attenuation (deficiencies in excess of 10 dB over 500 m) for frequencies in the range 5-10 kHz. These results have been explained with the aid of a statistical theory of finite-amplitude noise propagation. Detailed analysis of recordings from one test has provided a direct check on the nonlinear theory. Results from several different tests have been incorporated in a statistical model, which allows the nonlinear distortion of aircraft noise spectra to be estimated as a function of distance, level, spectrum shape, and atmospheric conditions. Preliminary results show encouraging agreement with the anomalous data.

Nomenclature

C_{pq} = cospectral density of two signals $p(t)$ and $q(t)$ defined as real part of S_{pq}
c = equilibrium sound speed
d = nozzle exit diameter
F = $[S_{p^2} - (4/3)C_{pp^3}]/p^4_{\rm rms}$
$f;\ f'$ = frequency; frequency moving with aircraft
G = $(2Q_{pz} - \beta C_{pp^2} - \alpha Q_{pp^2})/kp^3_{\rm rms}$
k = acoustic wavenumber ω/c
L = nonlinear correction to spectrum, Eq. (6)
P = nondimensional acoustic pressure $\Gamma p/\rho c^2$
$p(t)$ = pressure signal (zero mean value)
$p_\alpha(t)$ = Fourier transform of $\alpha'(\omega)\tilde{p}(\omega)$ with respect to ω
$\tilde{p}(\omega)$ = Fourier transform of pressure signal with respect to time
Q = $Q_{pp^2}/p^3_{\rm rms}$
Q_{pq} = quadspectral density of two signals $p(t)$ and $q(t)$, defined as imaginary part of S_{pq}
r = spherical radius
S = normalized power spectrum $S_p/p^2_{\rm rms}$
S_{pq} = cross-spectral density $(C_{pq} + iQ_{pq})$ of two signals $p(t)$ and $q(t)$
S_p, S_{p^2} = power spectral density of $p(t)$, $p^2(t)$
y = $\ln(r/r_0)$
$z(t)$ = second-order variable $p(t)p_\alpha(t)$
α = attenuation coefficient for linear waves
α' = $\alpha + i\beta$
β = dispersion coefficient for linear waves
Γ = nonlinearity coefficient, equal to $\frac{1}{2}(\gamma+1)$ for a perfect gas
ϵ = $\Gamma/\rho c^3$
ν = kinematic viscosity
ρ = fluid density
ω = radian frequency $2\pi f$

Subscripts

x, t' = partial derivatives with respect to indicated variable

I. Introduction

A. Statement of Problem

FLYOVER noise data presented in Ref. 1 suggest that nonlinear propagation effects can significantly modify the spectrum of aircraft noise, and consequently the perceived noise rating. Specifically, the measurements show that over distances of 500-1000 m, the high-frequency attenuation in the range 5-10 kHz may be only about half the amount predicted on the basis of linear propagation.[2-4] The effect occurs over distances typical of those used in noise certification measurements, and is of potential practical importance for two reasons.

First, it is standard practice to correct such measurements to prescribed atmospheric conditions without regard to the absolute noise level. If nonlinear effects are present, this procedure will give rise to errors.

Second, the importance of guaranteeing certification noise levels at an early stage means that accurate prediction of flyover noise is essential. If the perceived noise rating is influenced by nonlinear spectral distortion, it is important that the effect be quantified.

In this paper we review the relevant theory and present some recent experimental results relating to aircraft noise. A prediction scheme is then outlined, which allows the atmospheric attenuation of noise spectra to be estimated with allowance for nonlinear spectral distortion.

B. Background to Theory

Given the waveform of a propagating acoustic signal at one location, it is possible in principle to predict the waveform at other points further along the propagation path. For waves of moderate amplitude—typically below 165 dB in air—an approximate description based on Burgers' equation is appropriate, as explained by Lighthill[5] and Blackstock.[6] The approximation consists in adding a linear thermoviscous term to the exact Riemann equation for plane progressive waves in a compressible fluid and dropping terms of higher than second order in the wave amplitude. One form of the resulting equation is

$$p_x - \epsilon p p_{t'} = \tfrac{1}{2}\delta p_{t't'} \tag{1}$$

where the thermoviscous coefficient δ is

$$\delta = \left(\frac{\nu}{c^3}\right)\left(\frac{4}{3} + \frac{\gamma - 1}{Pr}\right)$$

and the nonlinear coefficient $\epsilon = \Gamma/\rho c^3$.

Fourier transformation of Eq. (1) from the time domain (t') to the frequency domain (ω) yields the equivalent description

$$\tilde{p}_x - \tfrac{1}{2} i\omega\epsilon\tilde{q} = -\tfrac{1}{2}\omega^2\delta\tilde{p} \tag{2}$$

in which $\tilde{q}(\omega)$ is the Fourier transform of $p^2(t)$. An advantage of working in the frequency domain is that Eq. (2) may be generalized to cover propagation in real gases with

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relaxation effects,[7] if the thermoviscous attenuation coefficient $\omega^2\delta/2$ is replaced by the complex quantity $\alpha'(\omega)=\alpha+i\beta$. Here α and β are the linear attenuation and dispersion coefficients for plane progressive waves, as determined by experiment.

Further generalization of Eqs. (1) and (2) to cover spherical and cylindrical outgoing waves (or propagation along a ray tube with arbitrary area variation) is straightforward provided the wavefront area varies slowly in the propagation direction.[5] Thus for spherical waves, the generalized frequency-domain equation is

$$\left(\frac{\partial}{\partial r}+\alpha'\right)r\tilde{p}=\tfrac{1}{2}i\omega\epsilon r\tilde{q} \tag{3}$$

Equation (3) is a valid description of nonlinear wave propagation provided: $\omega r/c \gg 1$ (quasi-plane wave propagation), $|\alpha'| \ll \omega/c$ (*weak attenuation and dispersion*), *and* $|p|/\rho c^2 \ll 1$ (weak nonlinearity).

Exact analytical solutions of these equations are available only for the original version Eq. (1) which refers to plane thermoviscous waves, although some progress has been made towards asymptotic solutions for the spherical wave problem.[8,9] To deal with the general case, a numerical scheme has been developed by Pestorius[10] which propagates the waveform a short distance using weak-shock theory,[11,12] and converts to the frequency domain to adjust for linear dispersion and attenuation before taking the next step in distance.

Pestorius's algorithm has been shown to agree well with laboratory experiments on plane waves in a long tube,[10,13] where attenuation and dispersion result from viscothermal effects at the tube well. Both single-frequency and broad-band excitation were used in these experiments. The algorithm has also been adapted by Theobald[14] to predict spherically spreading waves in air, with allowance for attenuation (but not dispersion) as given by Ref. 2; comparisons with an outdoor experiment using periodic signals again showed generally good agreement up to the maximum range of 76 m.

C. Statistical Description of Nonlinear Propagation

In the aircraft noise context, it is important to be able to predict noise spectra at any desired distance, given certain information on the noise near the source. Since the desired information is statistical, the input information should be statistical, but it must include more than the initial power spectrum; different waveforms with the same power spectrum may exhibit quite different amounts of nonlinear distortion. The additional information may be supplied either directly from experimental data, or via a statistical model which has some basis in experiment. Results obtained in both these ways will be presented in later sections.

Existing work on the nonlinear noise problem has been based on the statistical modeling approach, but with only a post hoc appeal to experimental data. The first step in this direction was taken by Pernet and Payne.[15] They devised a spectral distortion equation for noise signals by representing the generation of sum-frequency components as a square-law process, whose output statistics were modeled by assuming a Gaussian signal as input. A closely-related statistical model will be introduced in the present study in Sec. IV.

A more sophisticated approach to solving the spectral distortion problem is described by Rudenko and Soluyan.[16] Like Pernet and Payne, they give an explicit expression for the power spectrum at any distance, in terms of the power spectrum at some starting distance where the signal statistics are assumed Gaussian. However, their result is limited to media that are nondispersive and whose attenuation coefficient is independent of frequency, and it is not clear that their prediction can be modified to deal with more realistic situations.

Finally, Webster and Blackstock[17] have proposed a technique for reconstructing "statistically equivalent" waveforms from the power spectrum of a random noise signal, in order to permit the evolving power spectrum to be calculated as a function of distance. The waveform reconstruction involves assuming that the initial signal has a random phase distribution with respect to frequency; it was successfully used by Webster and Blackstock to reproduce the results of a plane-wave experiment in which the random input signal was Gaussian noise, whose phase distribution is indeed uniform. However, attempts to apply the same approach to outdoor noise propagation measurements[18,19] have proved disappointing so far. It appears that the random phase assumption may be unrealistic for non-Gaussian signals, particularly for signals which have already undergone some nonlinear distortion.

In contrast to the work mentioned above, we concentrate on developing a differential equation for the power spectrum as a function of distance. The intention is that the equation shall be solved numerically for whatever input signal and atmospheric conditions are encountered in practice, to yield the power spectrum at points either closer to or further from the source. Section II sets out the basic equations. Their experimental verification (using flyover noise measurements) is taken up in Sec. III. The last two sections deal with the prediction problem, and in particular the statistical modeling of the nonlinear terms; results are presented from a preliminary prediction program.

II. Theoretical Results

A. Spectral Distortion Equation

Over a wide range of frequencies, the propagation of small-amplitude sound waves in a uniform medium at rest is described by the linear equation $[\nabla^2+(k')^2]\tilde{p}=0$ with complex wavenumber

$$k'(\omega)=(\omega/c)-i\alpha' \qquad \text{(time factor } e^{i\omega t}\text{)} \tag{4}$$

The α' term allows for attenuation and dispersion in the medium, and is assumed to be small compared with ω/c. In particular, when the medium is air (at pressures and temperatures typical of the lower atmosphere), the preceding description of linear propagation is valid up to at least 100 kHz.

When nonlinear effects are included, and the sound field is assumed to consist of spherical outgoing waves, we arrive at Eq. (3). This is a model equation capable of describing aircraft noise propagation 1) at moderate intensities (levels below 165 dB re 20 μPa), 2) in the far field (at least one wavelength from the source), and 3) throughout the audio frequency range.

Since our objective is an equation for noise spectra, the deterministic equation [Eq. (3)] must be converted to statistical form. The equation is first multiplied through by r times the complex conjugate of $\tilde{p}$; ensemble averaging of the real part then gives

$$\frac{\partial}{\partial r}(r^2e^{2\alpha r}S_p)=-\omega\epsilon r^2e^{2\alpha r}Q_{pp^2} \tag{5}$$

where Q_{pp^2} is the imaginary cross-spectral density of the pressure and the pressure squared. In this equation the left-hand side would equal zero according to linear theory, and the Q_{pp^2} term on the right accounts for the nonlinear distortion of the spectrum which occurs during finite-amplitude propagation. Note that although β does not appear explicitly, its effect is implicit, in that the spectral transfer term Q_{pp^2} will be influenced by changes in waveform which arise from dispersion.

Equation (5) may be used to estimate nonlinear spectral distortion between two points on the propagation path. The quantities S_p and Q_{pp^2} are measured at one point, and the nonlinear distortion rate is deduced for that point on the path. Provided the other point is sufficiently close, variations in the distortion rate over the interval may be neglected. It is convenient for such purposes to write the equation in the normalized form

$$\frac{\partial L}{\partial y} = -krP_{\rm rms}Q/S \tag{6}$$

where

$$k=\omega/c_0, \qquad P=\Gamma p/\rho_0 c_0^2, \qquad \Gamma=(\gamma+1)/2$$

$$Q=Q_{pp^2}/p^3_{\rm rms}, \qquad S=S_p/p^2_{\rm rms}$$

$$L=\ln[(r^2e^{2\alpha r}S_p)/(r^2e^{2\alpha r}S_p)_0], \qquad y=\ln(r/r_0)$$

and subscript 0 denotes an arbitrary reference position.

The practical usefulness of Eq. (6) is limited, however, by the fact that Q/S may vary by a large factor over a long propagation path, and it is not sufficient simply to use a value measured near the source. Thus although the equation is useful in confirming the existence of nonlinear effects in situations where Q/S data are available (cf. Sec. III), the prediction problem remains unsolved. This point is discussed further subsequently.

B. Prediction Problem

Key requirements of a prediction scheme for estimating nonlinear spectral distortion are as follows.

1) The power spectrum S_p of the propagating signal should be predictable for any point on the propagation path, and any set of atmospheric conditions (pressure, temperature, relative humidity).

2) The required information concerning waveform statistics should be as simple as possible, i.e., the power spectrum S_p and the minimum number of additional parameters. (Note that knowledge of S_p alone is in principle insufficient, as the foregoing theory demonstrates.)

3) Statistical input information should, if possible, be limited to values measured near the source, to eliminate the influence of atmospheric variables.

In the light of requirement 3, Eq. (6) is unsuitable for prediction purposes because it involves a statistical parameter (Q/S) whose value changes in an unknown manner along the propagation path. As an example, if the signal statistics were Gaussian near the source, the third-order spectrum Q_{pp^2} would be zero initially, but would become finite as a result of nonlinear distortion once the signal had propagated a significant distance. Use of the initial value $Q/S=0$ for the whole propagation path would clearly be erroneous in this case, since it would result in zero spectral distortion.

With the aim of producing a prediction scheme for nonlinear noise propagation which meets the aforementioned criteria, a second spectral distortion equation has been developed,[20] essentially by differentiating Eq. (5) with respect to r. The result may be written in normalized form as

$$\frac{\partial^2 L}{\partial y^2} + \left(\frac{\partial L}{\partial y}\right)^2 = \frac{1}{S}(kr)^2(\tfrac{1}{2}FP^2_{\rm rms} + GP_{\rm rms}) \tag{7}$$

Here F and G are normalized spectra of fourth and third order, respectively, as defined in the Nomenclature.

The implementation of a prediction scheme based on Eq. (7), and in particular the estimation of the right-hand side for jet noise signals, are discussed in Sec. IV.

III. Experimental Confirmation of Nonlinear Theory

A. Evidence from Aircraft Flyover Data

Figure 1 shows the microphone arrangement for a flyover noise test program. A series of constant-power climbs was made with different turbojet aircraft at different thrust settings; brief details are given in Table 1. The noise was recorded at various points under the flight path, and comparative spectra were obtained for the same nominal source at different distances.

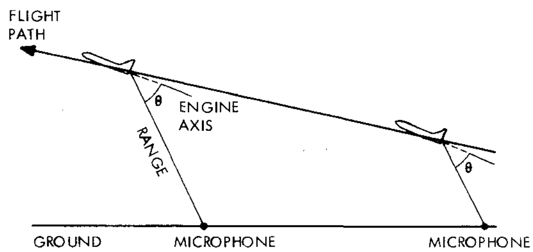


Fig. 1 Microphone arrangement for flyover noise tests; height of microphones above ground is 1.2 m.

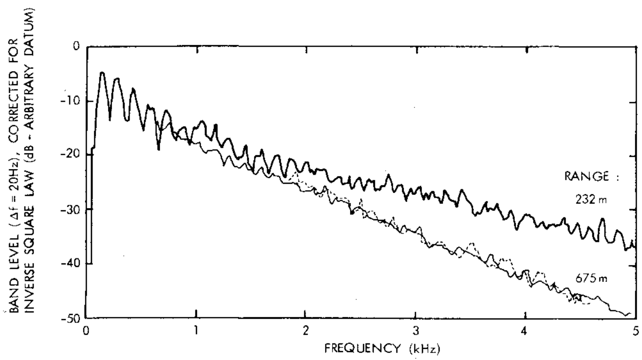


Fig. 2 Aircraft noise narrow-band spectra measured at the same angle and two different distances (test B - see Table 1); the broken curve at 675 m is a repeat analysis.

Table 1 Summary of tests analyzed

Test	A	B	C	D
Source	Four-engine aircraft, full power flyover	Two-engine aircraft Full power flyover	Two-engine aircraft Cutback flyover	Static engine, full power
Radiation direction (angle to exhaust axis), deg	60	60	60	67
Overall SPL at 20 m (re 20 μPa), dB	140	135	119	140
$f'd/U_J$ relative to source, at $f=1$ kHz	~1	0.8	1.1	0.8

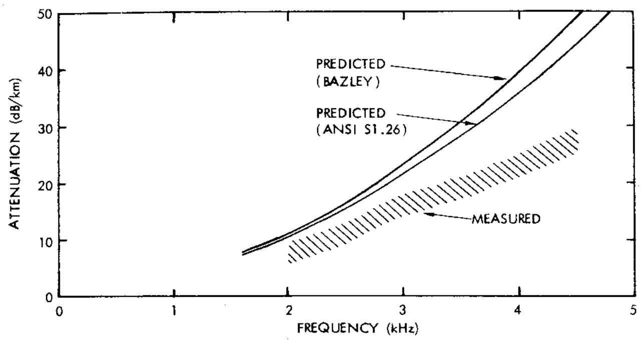


Fig. 3 Apparent atmospheric attenuation rate, deduced from Fig. 2, compared with standard predictions.

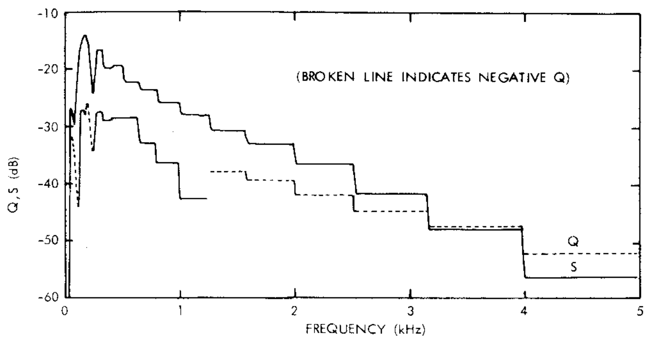


Fig. 4 Spectra of normalized quantities Q and S (test B, range 675 m); negative Q values are indicated by broken line.

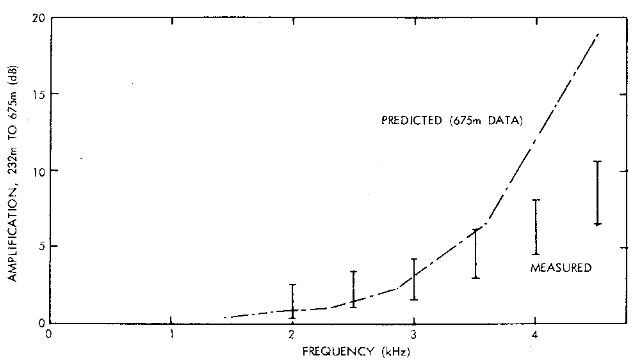


Fig. 5 Shortfall in high-frequency attenuation (from Fig. 3), compared with spectral transfer prediction from nonlinear theory (using statistical data measured at 675 m).

Figure 2 shows a typical comparison for a two-engined aircraft at full power; both spectra relate to the peak noise angle, approximately 60 deg to the jet axis. (Use of the peak noise angle for comparison minimizes the effect of timing errors. It also maximizes the effect of nonlinear distortion.) Subtraction of the two spectra gives the apparent atmospheric attenuation rate shown in Fig. 3, which at high frequencies falls substantially short of standard predictions based on linear propagation.[2,3] (Note that the linear attenuation predictions in Fig. 3 take proper account of the measured variations of temperature and humidity with height.)

In order to test whether nonlinear distortion could account for the anomalously low attenuation, the flyover noise signal at 675 m range was analyzed to give the spectral transfer factor Q [see Eq. (6)]. Values of Q and the normalized power spectrum S are plotted in Fig. 4; their ratio determines, according to Eq. (6), the local rate of attenuation (or amplification, if Q is negative) due to nonlinear processes. It is significant that Q does in fact become negative (as indicated by the broken line in Fig. 4) above 1.25 kHz, which covers the frequency range in Fig. 3 where a shortfall in attenuation is observed.

In Fig. 5 the amplification in dB, deduced from the measured spectra at 232 and 675 m, is compared with the predicted amplification based on spectral distortion theory. The order-of-magnitude agreement (within a factor of 2) indicates that the low attenuation values observed in Fig. 3 indeed are caused by nonlinear propagation. Note that the prediction in Fig. 5 is based on Q/S as measured at one point only, whereas ideally the comparison would involve an integral of Q/S along the propagation path. Furthermore, no attempt has been made in Fig. 5 to correct the 1.2-m microphone recordings for ground reflection, although Fig. 2 indicates the presence of a strong reflected signal at the lower frequencies.

B. Alternative Explanations of Measured Data

It is of course necessary to consider whether the attenuation anomaly revealed by Fig. 2 could be due to causes other than nonlinear propagation. Possible alternative explanations are as follows.

1) The spectrum measured at 675 m may be in error at high frequencies, owing to a combination of rapid high-frequency fall-off and finite filter bandwidth.

2) The aircraft conceivably did not act as a constant noise source, but for some reason produced more high-frequency noise at higher altitude.

3) The spectral levels at high frequencies may be artificially increased by nonlinearities in either the microphone or the tape recorder.

4) The high-frequency levels are possibly augmented by background acoustic or electronic noise, particularly in the 675 m recording (where the absolute levels were lower).

We discuss these possibilities in turn next. The flyover tests and data analysis procedures are described in detail in Ref. 20.

1) This point would be relevant to ⅓ octave measurements, but the 20 Hz bandwidth spectra in Fig. 2 provide ample frequency resolution (digital analysis using 50 ms block length and Hanning window).

2) The absolute discrepancies in attenuation (approaching 10 dB at 5 kHz) are too large to be accounted for by any plausible variation ($\pm 5\%$) in nozzle exit temperature or jet velocity during climb. Jet noise predictions have been run that show a sensitivity of less than 1 dB in spectrum shape over this range. Moreover Fig. 2 shows that below 500 Hz, the levels corrected for spherical spreading are indistinguishable (within 1 dB), indicating a constant low-frequency source output.

3) The overall sound pressure levels were 104 dB at 675 m, and 114.5 dB at 232 m (both re 20 μPa). A pistonphone recording (124 dB at 250 Hz) was analyzed from the same tape; the spectrum showed a high-frequency floor level, in the range 2-5 kHz, some 63 dB below the peak. Both the aircraft and pistonphone signals were recorded on tape at approximately the same voltage level (within 1 dB). The pistonphone test, therefore, indicates a linear dynamic range of over 60 dB for the spectra in Fig. 2, which more than covers the data.

4) The same test confirms the absence of significant background noise from the tape recorder or analysis system, over the frequency range of Fig. 2 (up to 5 kHz). Acoustic background noise data are not available; however, the lowest measured level in the 5 kHz ⅓-octave band was 52 dB (at 675 m), which is well above the likely noise background.

Our conclusion is that none of the alternatives considered is able to account for the anomalous attenuation measurements of Fig. 3. On the other hand, the hypothesis of nonlinear spectral distortion does provide a reasonable explanation of the data, in that the shortfall in attenuation is predicted within a factor of 2 by Eq. (6). Further data of the type shown in Figs. 2 and 3, from a large number of flyover tests, have been assembled by Gallard and Gower[1] in support of the nonlinear propagation hypothesis.

IV. Prediction Scheme for Finite-Amplitude Noise Propagation

A. Statistical Modeling of Spectral Distortion Terms

We note that for Gaussian signals, since odd-order statistics vanish but even-order statistics in general do not,

$$G=0 \text{ but } F\neq 0 \tag{8}$$

Thus the right-hand side of Eq. (7) may be modeled in a nontrivial manner by assuming a quasinormal, or Gaussian, relationship between these higher-order statistics and the ordinary power spectrum of the signal.

Specifically, if the joint probability distribution of $p(t)$ and $p(t+\tau)$ were normal for all values of the time delay τ, the fourth-order spectral quantity F would be related to S by[21]

$$F=2T-4S \qquad (\omega\neq 0) \tag{9}$$

Here,

$$T=\int_{-\infty}^{\infty} S(\omega')S(\omega-\omega')\mathrm{d}\omega' \tag{10}$$

is the convolution of S with itself. A signal which obeys Eq. (9) will be called quasinormal.

B. Experimental Test of the Quasinormal Hypothesis for Jet Noise

Several aircraft flyover and static engine noise recordings have been analyzed to yield values of F, T, and S (see Ref. 20 for details). The signal in each case was dominated by jet noise. We may test the validity, or otherwise, of treating the signal as quasinormal for purposes of Eq. (7) by plotting the ratio $(2T-4S)/F$ as a function of frequency.

Some typical results are shown in Fig. 6. Note that the ratio equals unity if the signal is quasinormal. For each of the three tests, the measured ratios are close to unity over the main energy-containing part of the spectrum (100-800 Hz), but fall to ½ or less at the upper end of the measured range (4-5 kHz). There is an intermediate range around 1-2 kHz where both $(2T-4S)$ and F change sign, with consequent large variations in their ratio.

It appears likely that a better approximation of F in the high-frequency region, where spectral distortion effects become important, would be

$$F=2(T-BS) \tag{11}$$

with B somewhat less than 2. Some preliminary predictions based on Eq. (11) are compared with flyover measurements in Sec. V.

C. Difficulty with the Quasinormal Hypothesis

Use of Eq. (7), with F given by Eq. (9) and the G term neglected, has been found to lead to negative "dropouts" in spectra predicted for sufficiently large distances. (A similar feature has been noted in homogeneous turbulence theory.[22]) Such dropouts can be avoided either by using Eq. (11) for F with a sufficiently small value of B (less than 1), or by not allowing the value of Q/S implied by Eq. (6) to exceed some limit of order 1.

If the latter method is used, Eq. (7) is overridden in the relevant part of the spectrum as soon as it predicts a Q/S value in excess of the limit. The calculation then proceeds with Q/S held constant until Eq. (7) predicts a decrease in Q/S, at which point it is reinstated. This procedure was used in obtaining the predictions shown subsequently. Note that it is invoked only in the peak region of the spectrum, where Q/S is positive, i.e., nonlinear attenuation occurs.

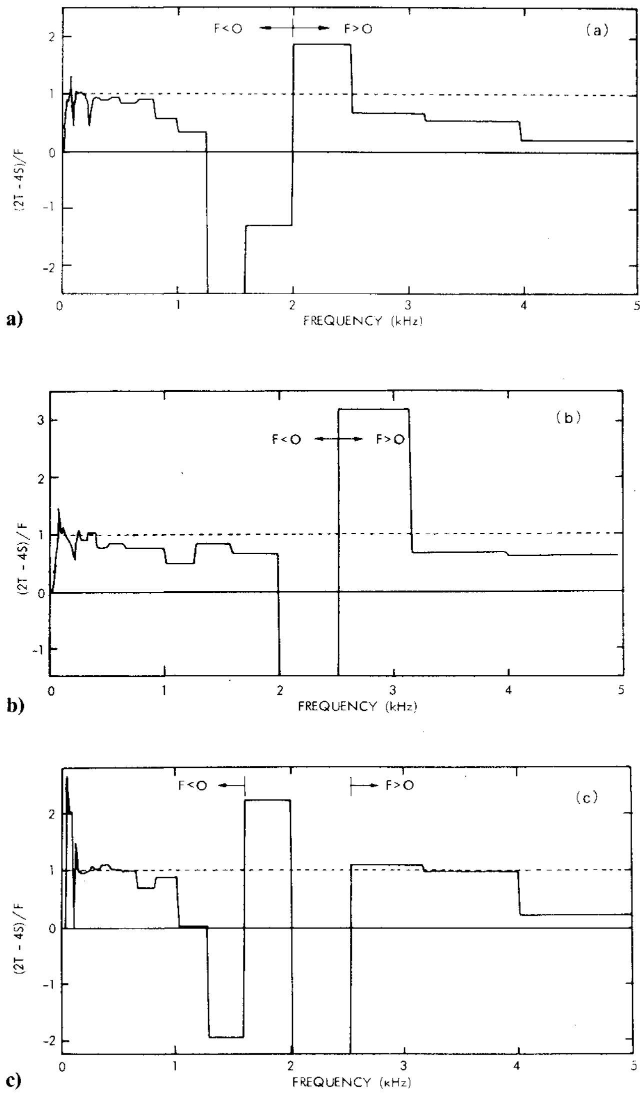


Fig. 6 Test of quasinormal hypothesis: a) test B, 675 m; b) test C, 535 m; c) test D, 33 m (static engine).

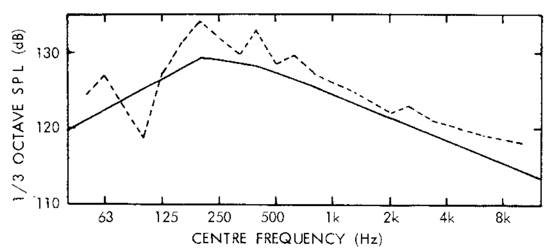


Fig. 7 Derivation of initial spectrum at 20 m from measured spectrum at 262 m (test A): ----, measured spectrum corrected for spherical spreading to 20 m; ——, derived free-field 20 m spectrum.

V. Preliminary Attempts at Prediction

A. Starting Conditions

The method of calculating nonlinear noise propagation just described requires starting values for both the noise spectrum S_p and the derivative $\partial L/\partial y$ at $y=0$. Since no a priori information is available on the latter, beyond the hypothesis that $\partial L/\partial y$ vanishes close to the source (Gaussian source assumption), we limit ourselves to predictions that start close to the source and work outward.

The predictions presented in this section relate to flyover test A (Table 1). Figure 7 shows the starting spectrum assumed at a radius of 20 m. This was obtained by ex-

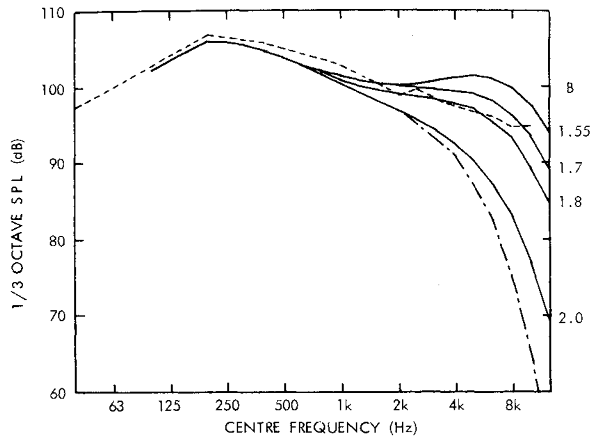
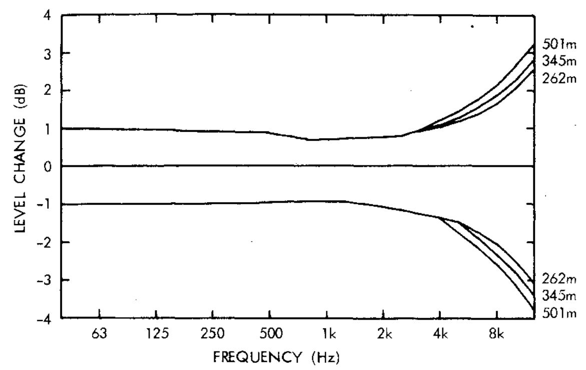


Fig. 8 Comparison of prediction with measurement at 262 m (test A): ——, nonlinear predictions for different values of B; ----, measured spectrum (corrected for ground reflection); — · — · —, linear prediction.



ig. 9 Level change produced by raising or lowering the starting pectrum in Fig. 7 by 1 dB (test A).

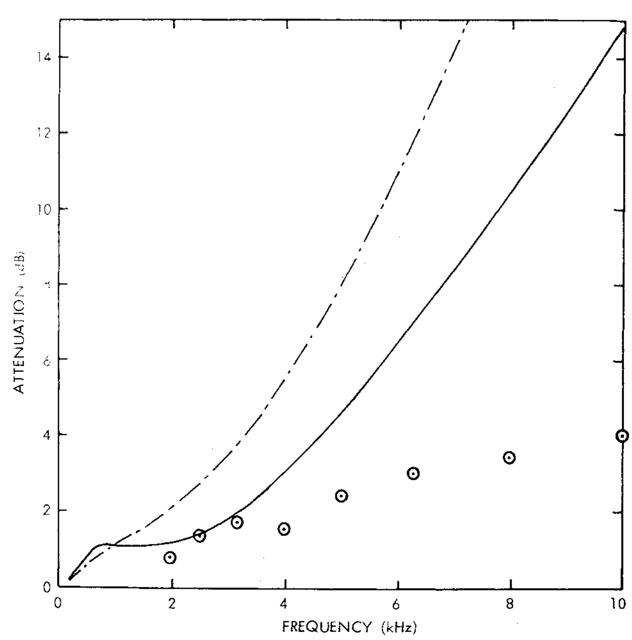


Fig. 10 Measured attenuation of aircraft noise between 262 and 501 m (test A,I) in ⅓-octave bands as a function of center frequency; predictions: — — — (linear), —— (nonlinear).

trapolating back from the smallest measurement distance, with no allowance for attenuation, to establish the spectrum up to $f = 1$ kHz. The high frequency fall off of the ⅓-octave source spectrum was then taken as 1 dB/⅓ octave in line with jet noise predictions. An allowance has been made for the ground reflection present in the measured spectra, since the prediction model refers to free-field propagation.

B. Value of the Parameter B

In order to establish the best value of B [Eq. (11)] for prediction purposes, noise spectra at the first measurement distance in test A were predicted for a range of B values, starting from the free-field 20 m spectrum in Fig. 7. (The choice of 20 m as starting distance has been shown not to be critical; the predictions vary by less than ½ dB, for any choice between 20 and 40 m.)

Figure 8 shows the various predictions compared with the measured spectrum (corrected for ground reflection). The predicted high-frequency levels are quite sensitive to B; a value of between 1.7 and 1.8 gives the best agreement. Subsequent predictions in this paper accordingly use $B = 1.75$.

C. Sensitivity to Changes in Level

It is clear from Fig. 8 that the measured flyover spectrum at 262 m deviates markedly above 2 kHz from the linearly predicted spectrum. Our prediction scheme attributes the deviation to the transfer of energy from low to high frequencies. Since the effect is nonlinear, any absolute error in the starting spectrum level will influence the predictions, and hence, in principle, the choice of B. The sensitivity of the predicted spectra to small changes in absolute level has therefore been examined, using the test A spectrum.

Figure 9 shows that for propagation between 20 and 262 m, the present method predicts an almost linear relationship up to 2 kHz, with significant nonlinearity becoming apparent at higher frequencies. This is consistent with Fig. 8, where the nonlinear and linear spectrum predictions stay within 4 dB up to 2 kHz and then diverge rapidly. The important conclusion to be drawn from Fig. 9, however, is that even up to 8 kHz the spectrum shape as predicted at 262 m is not unduly sensitive to the initial signal level. A 1 dB error in the 20 m starting spectrum translates, at 8 kHz, into an error slightly less than 2 dB at 262 m; the 8 kHz attenuation error is less than 1 dB. Thus, the estimate $B = 1.75$ originally obtained from Fig. 8 is not vulnerable to modest errors in absolute level, of order ± 1 dB.

At distances beyond 262 m, the predictions in the nonlinear region of Fig. 9 become increasingly inaccurate. The shortcomings of the current prediction scheme at large distances are demonstrated in the following section.

D. Long-Range Attenuation: Comparison of Predictions with Data

The nonlinear prediction scheme has been shown successful in reproducing the measured spectrum at 262 m in test A, up to a frequency of 8 kHz (Fig. 8). We now wish to see whether it can predict the measured attenuation between two test points in the same flyover series (262 and 501 m from the aircraft).

Rather than use single-point data for this purpose, we have taken advantage of test data at several distances by drawing a smooth curve through all the available points, plotted as ⅓-octave level vs distance for each frequency. The drop in level between 262 and 501 m, adjusted for spreading, then defines the measured attenuation values in Fig. 10.

Plotted as a solid line in Fig. 10 is the predicted attenuation curve, based on the current nonlinear model with modified Gaussian statistics. It follows the measurements closely up to 3.15 kHz and then diverges upward. Evidently the success of the nonlinear propagation model between 20 and 262 m (Fig. 8) is not maintained between 262 and 501 m, although the model goes part way toward closing the gap between measurements and linear theory.

Two further comments on Fig. 10 are relevant. First, the ground reflection problems encountered in Fig. 8 are largely eliminated because the levels measured at two microphones of the same height are subtracted to give the attenuation. Second, at high frequencies and large distances it is necessary to take account of the actual ⅓-octave filter shape when predicting ⅓-octave attenuations[23]; in Fig. 10 the difference is probably small (an ideal rectangular filter was actually used for the predictions), but this point needs to be checked wherever ⅓-octave analysis has been performed on steeply sloping spectra.

VI. Concluding Remarks

1) Nonlinear propagation, with consequent cumulative distortion of the acoustic waveform, has been shown to account for the anomalously low attenuation of high-frequency noise observed in certain aircraft flyover tests (Sec. III).

2) In one case described (test A), the shortfall in attenuation over a distance of approximately 500 m amounted to 30-50 dB in the frequency range 8-10 kHz.

3) A numerical prediction scheme has been developed to provide estimates of nonlinear attenuation in a uniform still atmosphere. The scheme in its present form requires empirical input information concerning the bivariate statistics of the noise signal.

4) For aircraft noise dominated by jet noise, some information is available on the relevant statistics. This has been incorporated in a statistical model, and fed into the prediction scheme to yield comparisons with aircraft noise measurements.

5) Preliminary tests are encouraging, although the attenuation is still overpredicted at high frequencies. Loss of accuracy occurs at progressively lower frequencies with increasing range; e.g., typically 8-10 kHz at 250 m, but 3-4 kHz at 500 m.

6) Further work is required to refine the nonlinear prediction program in the following areas: a) Improved statistical modeling of terms in the spectral distortion equation, to overcome the high-frequency limitation; and b) incorporation of a layered atmosphere to allow for variations of pressure, temperature, and humidity along the propagation path.

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